

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

## MATHEMATICS

9709/02
Paper 2 Pure Mathematics 2 (P2)

Additional Materials: | Answer Booklet/Paper |
| :--- |
| Graph Paper |
| List of Formulae (MF9) |

## READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet. Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50 .
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

1 Solve the inequality $|3 x-1|<2$.

2 Use logarithms to solve the equation $4^{x}=2\left(3^{x}\right)$, giving your answer correct to 3 significant figures.

3 Find the exact value of $\int_{0}^{\frac{1}{6} \pi}(\cos 2 x+\sin x) d x$.

4 The polynomial $2 x^{3}+7 x^{2}+a x+b$, where $a$ and $b$ are constants, is denoted by $\mathrm{p}(x)$. It is given that $(x+1)$ is a factor of $\mathrm{p}(x)$, and that when $\mathrm{p}(x)$ is divided by $(x+2)$ the remainder is 5 . Find the values of $a$ and $b$.

5 (i) Express $5 \cos \theta-\sin \theta$ in the form $R \cos (\theta+\alpha)$, where $R>0$ and $0^{\circ}<\alpha<90^{\circ}$, giving the exact value of $R$ and the value of $\alpha$ correct to 2 decimal places.
(ii) Hence solve the equation

$$
\begin{equation*}
5 \cos \theta-\sin \theta=4 \tag{4}
\end{equation*}
$$

giving all solutions in the interval $0^{\circ} \leqslant \theta \leqslant 360^{\circ}$.

6 It is given that the curve $y=(x-2) \mathrm{e}^{x}$ has one stationary point.
(i) Find the exact coordinates of this point.
(ii) Determine whether this point is a maximum or a minimum point.

7 The equation of a curve is

$$
x^{2}+y^{2}-4 x y+3=0
$$

(i) Show that $\frac{\mathrm{d} y}{\mathrm{~d} x}=\frac{2 y-x}{y-2 x}$.
(ii) Find the coordinates of each of the points on the curve where the tangent is parallel to the $x$-axis.

8 The constant $a$, where $a>1$, is such that $\int_{1}^{a}\left(x+\frac{1}{x}\right) \mathrm{d} x=6$.
(i) Find an equation satisfied by $a$, and show that it can be written in the form

$$
\begin{equation*}
a=\sqrt{ }(13-2 \ln a) \tag{5}
\end{equation*}
$$

(ii) Verify, by calculation, that the equation $a=\sqrt{ }(13-2 \ln a)$ has a root between 3 and 3.5.
(iii) Use the iterative formula

$$
a_{n+1}=\sqrt{ }\left(13-2 \ln a_{n}\right)
$$

with $a_{1}=3.2$, to calculate the value of $a$ correct to 2 decimal places. Give the result of each iteration to 4 decimal places.

BLANK PAGE

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

University of Cambridge International Examinations is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.

